This question paper contains 8 printed pages]

Roll No. 2019

(21)

S. No. of Question Paper : **7482**

Unique Paper Code : **32221102-OC**

Name of the Paper : **Mechanics**

Name of the Course : **B.Sc. (Hons.) Physics**

Semester : **I**

J

Duration : **3 Hours**

Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *five* questions in all.

Question No. **1** is compulsory.

All questions carry equal marks.

Use of non-programmable scientific calculator is allowed.

1. Attempt any *five* of the following questions :

   (*i*) Show that in the absence of external forces, the velocity of the centre of mass remains constant.

   (*ii*) A body of radius R and mass m is rolling horizontally without slipping with speed v. It then rolls up a hill to a maximum height h. If $h = 3v^2/4g$, what is the moment of inertia of the body ?

(*iii*) Show that a two body problem involving central force can always be reduced to a form of a one-body problem.

(*iv*) A particle of mass m moving along a path given by $\vec{r} = a \cos \omega t \, \hat{i} + b \sin \omega t \, \hat{j}$. Calculate the angular momentum about the origin.

(*v*) Find the centre of mass of a thin rod of length l whose density varies with distance x from one end as : $\rho = \rho_0 x^2 / l^2$ where ρ_0 is a constant.

(*vi*) A 6000 kg rocket is set for vertical firing. If the gas exhaust speed is 1000 m/s, how much gas must be ejected each second to supply the thrust needed to overcome the weight of the rocket ?

(*vii*) Show that $\Delta s^2 = c^2 \Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2$ is a Lorentz invariant.

(*viii*) Two spaceships approach each other, each moving with the same speed as measured by a stationary observer on the Earth. Their relative speed is $0.7c$. Determine the velocities of each spaceship as measured by the stationary observer on Earth. 5×3=15

2. (*a*) A particle is projected from the top of a tower of height h with a velocity u at an angle α to the horizontal. Show that its range on a plane through the point of projection inclined at an angle θ below the horizontal through the point of projection is given by :

$$R = \frac{2u^2 \cos \alpha}{\cos^2 \theta} \sin (\alpha + \theta)$$ 7

(*b*) A particle of mass m moves under a conservative force with potential energy $V(x) = cx/(x^2 + a^2)$, where c and a are positive constants. Find the position of stable equilibrium and the period of small oscillations about it. 5

(*c*) Show that inertial mass and gravitational mass are proportional to each other. 3

3. (a) A particle of mass m_1 moving with velocity v_1 collides elastically with a particle of mass m_2 at rest in the laboratory frame. The scattering angle of mass m_1 as measured in laboratory frame is ϕ and the scattering angle of it as measured in C. M. frame is θ. Discuss the behavior of particle of mass m_1, giving proper geometric construction, when : 6

(i) $m_1 = m_2$

(ii) $m_1 >> m_2$

(iii) $m_1 << m_2$.

(b) Object A with mass m_1 is initially moving with a speed v_{1i} = 3.0 m/s and collides elastically with object B that has the same mass, $m_2 = m_1$ and is initially at rest. After the collision, object A moves with an unknown speed v_{1f} at an angle $\theta_{1f} = 30°$ with respect to its initial direction of motion and object B moves with an

unknown speed v_{2f} at an unknown angle θ_{2f} w.r.t the initial direction of motion of m_1. Find the final speeds of each of the objects and the angle θ_{2f}. 9

4. (a) Discuss the energy diagram for the planetary motion for all possible values of energy. 5

(b) Consider the motion of a particle of mass m under the influence of a force $\vec{F} = -k\vec{r}$, where k is a positive constant and $\vec{r}$ is the position vector of the particle :

(i) Prove that the motion of the particle lies in a plane.

(ii) Find the position of the particle as function of time, assuming that at $t = 0$, $x = a$, $y = 0$ and the velocity components $v_x = 0$, $v_y = v$.

(iii) Show that the orbit is an ellipse.

(iv) Find the period of motion of the particle.

(v) Does the motion of the particle obey Kepler's laws of planetary motion ? 7

(c) Prove that the angular momentum is conserved under the action of a central force. 3

5. (a) Establish the equation of motion of a damped harmonic oscillator explaining each term clearly. Solve the same for lightly damped case. 7

(b) Show that the average kinetic energy of a particle performing simple harmonic motion is equal to its average potential energy. 5

(c) The quality factor Q of a tuning fork is 5.0×10^4. Calculate the time-interval after which its energy becomes (1/10)th of its initial value. The frequency of the fork is 300 s^{-1}. (take $\log_e 10 = 2.3$) 3

6. (a) Consider the earth to be a sphere of radius R having angular speed ω. Prove that :

(i) The effective value of 'g' at latitude λ is given by $g_{eff} = g_0[1 - (2x - x^2) \cos^2 \lambda]^{1/2}$ where g_0 is the true acceleration due to gravity and $x = \omega^2 R/g_0$.

(ii) If $x << 1$; then $g_{eff} = g_0 - \omega^2 R \cos^2 \lambda$. 7,2

(b) Consider a body dropped from a height of 10 m, at a latitude of 30° N. Find, approximately, the horizontal deflection due to the Coriolis effect when it reaches the ground. Neglect air resistance. 3

(c) Calculate the fictitious force and the total force acting on a body of mass 5 kg relative to a frame moving with a downward acceleration of 2 m/s^2. 3

7. (a) Discuss how the null result of the Michelson-Morley experiment was explained. 5

(b) State the postulates of Einstein's special theory of relativity. Derive the Lorentz space-time transformation equations. Show that for the values of $v<<c$, Lorentz transformations reduces to the Galilean transformations. 8

(c) Explain space-like and time-like intervals. 2

8. (*a*) Why is classical expression for the kinetic energy not applicable in relativistic region ? Prove that relativistic kinetic energy is given by the relation $E_k = (m - m_0)c^2$. Also show that if $v \ll c$, it leads to the classical expression for kinetic energy. 8

(*b*) Two light sources A and B situated 10 meters apart flash light signals at an interval of one nanosecond. At what time interval will an observer travelling at a speed of $0.9c$ along the direction AB sees the two events ? 5

(*c*) What is a massless particle ? Give *two* examples. 2

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[This question paper contains 4 printed pages]

Your Roll No. : 2019

Sl. No. of Q. Paper : 8599 J

Unique Paper Code : 32221101

Name of the Course : B.Sc. (Hons.) Physics

Name of the Paper : Mathematical Physics-I

Semester : I

Time : 3 Hours ***Maximum Marks : 75***

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) Question **NO.1** is compulsory.

(c) Attempt **four** more questions out of the rest.

(d) Non-programmable calculators are allowed.

1. Do any **five** of the following : 5×3=15

(a) Determine the linear independence/linear dependence of e^x, xe^x, x^2e^x.

(b) Determine the order, degree and linearity of the following differential equation.

$$\frac{d^3y}{dx^3} + x^2\left(\frac{d^2y}{dx^2}\right)^2 = 0$$

(c) Find the are of the triangle having vertices at P (1,3,2), Q (2,-1,1) and (-1,2,3).

(d) Let $\vec{A}$ be a constant vector. Prove that

$$\vec{\nabla}\left(\vec{r}.\vec{A}\right) = \vec{A}$$

P.T.O.

(e) Find the acute angle between the surfaces $xy^2z-3x-z^2=0$ and $3x^2-y^2+2z=1$ at the point $(1,-2,1)$

(f) A random variable X has probability density function

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-(x-5)^{2/2}}$$

$-\infty < x < \infty$

Find the mean

2. (a) Solve the simultaneous differential equations given below.

$$\frac{dy}{dt} = y \ , \qquad \frac{dx}{dt} = 2y + x$$

(b) Two independent random variables X and Y have probability density functions $f(x) = e^{-x}$ and $g(y)=2e^{-2y}$ respectively. What is the probability that X and Y lie in the intervals $1<x\leq 2$ and $0 < y \leq 1$

The time rate of change of the temperature of a body at an instant t is proportional to the temperature difference between the body and its surrounding medium at that instant.

(c) Box A contains 8 items out of which 3 are defective. Box B contains 5 items out of which 2 are defective. An item is drawn randomly from each box. 5+5+5

(i) What is the probability that both the items are non-defective ?

(ii) What is the probability that only one item is defective ?

(iii) What is the probability that the defective item came from box A ?

3. Solve the following differential equations.

(a) $y'' + y = \sec x$ 8

(b) $(z + ye^{xy})dx + (xe^{xy} - 2y)dy = 0$ 7

4. (b) Solve the initial value problem. 8

(i) $y'' + 4y' + 8y = \sin x$

(ii) $y(0) = 1 \ , \ y'(0) = 0$

(b) A metal bar at a temperature 100^0 F is placed in a room at a constant temperature of 0^0F. After 20 minutes the temperature of the bar is 50^0 F. Find : 7

(i) The time it will take the bar to reach a temperature of 25^0 F

(ii) Temperature of the bar after 10 minutes

5. (a) If v denotes the region inside the semicircular cylinder

$$0 \leq x \leq \sqrt{a^2 - y^2} \quad 0 \leq z \leq 2a$$

Evaluate $\iiint_v x\,dv$ 7

(b) 17 8

6. (a) Find the directional derivative of $\varphi = 4xz^3 - 3x^2y^2z$ at $(2,-1,2)$ in the direction $2\hat{i} - 3\hat{j} + 6\hat{k}$ 5

(b) Find the value of $\nabla^2(\ln r)$

(c) Prove that : 5

$$\iiint \frac{dv}{r^2} = \oint\!\!\oint_S \frac{\bar{r}.\hat{n}}{r^2}\, ds$$

Where v is the volume of region enclosed by surface

7. (a) Suppose $\vec{A} = (2y + 3)\hat{i} + xz\hat{j} + (yz - x)\hat{k}$

Evaluate $\int_c \vec{A}.d\vec{r}$ along the following paths : 9

(i) $x = 2t^2$, $y = t$, $z = t^3$ from $t = 0$ to $t = 1$

(ii) The straight line from (0,0,0) to (0,0,1) then to (0,1,1) and then to (2,1,1)

(iii) The straight line joining (0,0,0) and (2,1,1)

(b) Evaluate $\iint \vec{A}.\hat{n}\, dS$ where $\vec{A} = z\hat{i} + x\hat{j} - 3y^2z\hat{k}$ and S is the Surface of the cylinder $x^2+y^2=16$ included in the first octant between $z = 0$ to $z = 5$ 6

This question paper contains 8+2 printed pages]

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S. No. of Question Paper : **8619** 23

Unique Paper Code : **32221102** J

Name of the Paper : **Mechanics**

Name of the Course : **B.Sc. (Hons.) Physics**

Semester : **I**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Question No. **1** is compulsory and carriers **19** marks.

Answer any *four* of the remaining six, each carrying **14** marks, attempting any *two* parts out of three from each question.

1. Attempt all parts of this question :

(*i*) Calculate the percentage contraction of a rod moving with a velocity 0.8c in a direction inclined at 45° to its own length. 3

(*ii*) A particle slides back and forth on a frictionless track whose height as a function of horizontal position x is given by $y = ax^2$, where $a = 0.92\ m^{-1}$. If the particle's maximum speed is 8.5 m/s, find the turning points of its motion. 3

P.T.O.

(*iii*) A space traveller weighs 80 kg on earth. Find the weight of the traveller on another planet whose radius is twice that of the earth and whose mass is 3 times that of the earth. 3

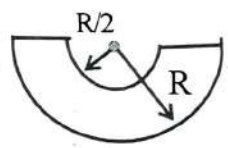
(*iv*) A rigid body is rotating about its axis of symmetry, its moment of inertia about the axis of rotation being 1 kg m^2 and its rate of rotation 2 rev/s. What is its angular momentum about the given axis ? What additional work will have to be done to double its rate of rotation ? 3

(*v*) A particle, moving in a straight line with S.H.M. of period $2\pi/\omega$ about a fixed point O, has a velocity $\sqrt{3}b\omega$ when at a distance b from O. Calculate its amplitude and the time it takes to cover the rest of its distance. 3

(*vi*) A 4800 kg elephant is standing at one end of a 15000 kg rail car, which is at rest all by itself, on a frictionless horizontal track. The elephant walks 19 m towards the other end of the car. How far does the car move ? 4

2. (*i*) (*a*) Find the location of the center of mass of a solid hemisphere of uniform density and radius R.

(*b*) Mass in the shape of a hemisphere of radius R/2 is removed from the hemisphere in part (*a*), as shown in the figure. Where is the center of mass of the remaining mass ? 4+3



(*ii*) Two particles having masses m_1 and m_2 move so that their relative velocity is v and the velocity of their centre of mass is v_{cm}. Prove that the total kinetic energy of the system is $(Mv_{cm}^2 + \mu v^2)/2$, where M is the total mass and μ is the reduced mass of the system. 7

(*iii*) An empty freight car of mass 500 kg starts from rest under an applied force of 100 N. At the same time sand begins to run into the car at a steady rate of

20 kg/s from a hopper at rest on the track. Find the speed of the car when 100 kg of sand has been transferred. 7

3. (*i*) Obtain an expression for the moment of inertia of a solid cylinder about an axis through its centre and perpendicular to its axis of cylindrical symmetry. 7

(*ii*) A ring of mass 0.3 kg and radius 0.1 m and a solid cylinder of mass 0.4 kg and of the same radius are given the same kinetic energy and released simultaneously on a flat horizontal surface such that they begin to roll as soon as released towards a wall which is at the same distance from the ring and the cylinder. Assuming that the rolling friction in both cases is negligible, find out which object reaches the wall first ? 7

(*iii*) A uniform rod of mass M and length L lies on a smooth horizontal plane. A particle of mass m moving at a speed v perpendicular to the length of the rod strikes it a distance L/4 from the centre and stops after the collision. Find :

(*a*) The velocity of the centre of the rod.

(*b*) The angular velocity of the rod about its centre just after collision. 4+3

4. (*i*) Derive the expression for the gravitational potential due to a spherical shell of radius R and mass M at a point outside the shell and also at a point inside the shell. Give its graphical representation. 7

(*ii*) A bead of mass m slides without friction on a smooth rod along the x-axis. The rod is equidistant between two spheres of mass M. The spheres are located at $x = 0$, $y = \pm a$ and attract the bead gravitationally :

(*a*) Find the potential energy of the bead.

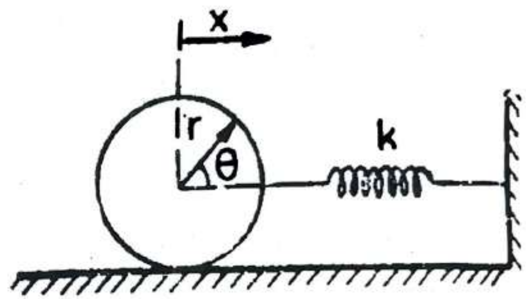
(b) The bead is released at $x = 3a$ with velocity v_0 towards the origin. Find the speed as it passes the origin.

(c) Find the frequency of small oscillations of the bead about the origin. 3+2+2

(iii) A particle of mass m moves in the central force field with the force function $f(r) = -Kr$, with $K > 0$. Find the effective potential energy and hence show that all the orbits are bounded. Find the radius and period of circular orbits, if any. 7

5. (i) What do you understand by 'logarithmic decrement', 'relaxation time' and 'quality factor' of a weakly damped harmonic oscillator ? Show that the average energy of a weakly damped harmonic oscillator decays exponentially with time. 3+4

(ii) A circular solid cylinder of radius r and mass m is connected to a spring of spring constant k as shown in the figure below.



Determine the frequency of horizontal oscillations of the system if the cylinder :

(a) Slips on the surface without rolling.

(b) Rolls on the surface without slipping.

Neglect friction. 3+4

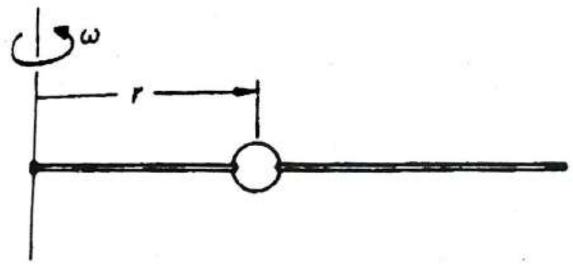
(iii) A particle of mass m with velocity v_0 collides elastically with another particle of mass M at rest, and is scattered through angle θ in the centre of mass frame. Show that the final velocity of mass m in the laboratory frame is :

$$v_f = \left(\frac{v_0}{m+M}\right)(m^2 + M^2 + 2mM\cos\theta)^{1/2}$$

Also find the fractional loss of kinetic energy of mass m if $m = M$. 7

6. (*i*) How does the rotation of Earth about its axis affect the acceleration due to gravity experienced by a body at rest at a point on the surface of earth ? Support your answer with a suitable derivation and diagram. 7

(*ii*) A bead of mass 'm' slides without friction on a rigid wire rotating at constant angular speed ω as shown in the figure. Find an expression for the force exerted by the wire on the bead that is initially at rest at a distance r_0 from the axis. 7



(*iii*) The space and time coordinates of two events as measured in frame S are :

Event 1 : $x_1 = x_0,\ t_1 = x_0/c,\ y_1 = z_1 = 0,$

Event 2 : $x_2 = 2x_0,\ t_2 = x_0/c,\ y_2 = z_2 = 0.$

Find the velocity of another frame S' in which the second event occurs by time $x_0/2c$ before the first event. 7

7. (*i*) Derive the expression for relativistic Doppler's effect. 7

(*ii*) A particle with a rest mass m_0 and kinetic energy $3m_0c^2$ makes a completely inelastic collision with a stationary particle of rest mass $2m_0$, without any radiation loss and the two particles forming a composite particle. What is the rest mass of the composite particle and its speed ? 7

(*iii*) (*a*) Suppose that a particle moves relative to O' with a constant velocity of $c/2$ in the $x'y'$-plane such that its trajectory makes an angle of 60° with the x'-axis. If the velocity of O' with respect to O is $0.6c$ along the x-x'-axis, find the equations of motion of the particle as determined by O.

(*b*) Define proper time. What is time dilation ? With what velocity should a rocket move so that as observed from Earth every year spent on the rocket corresponds to 4 years on Earth ? 4+3